2019 8th International Conference on Modern Circuits and Systems Technologies (MOCAST)

Performance Comparison of Companding-Based PAPR Suppression Techniques in OFDM Systems

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***Abstract*—The suppression of high peak-to-average-power-ratio (PAPR) values is crucial to the performance of orthogonal frequency division multiplexing (OFDM) systems. High PAPR gives rise to the non-linear clipping-induced harmonic distortions which degrade the end-to-end bit error rate performance. This paper presents a comparative analysis for various companding-based PAPR reduction techniques. Simulation results provide useful guidelines for the design and development of power and bandwidth efficient OFDM systems.**

***Index Terms*—Companding techniques, orthogonal frequency division multiplexing (OFDM) and peak-to-average-power-ratio (PAPR).**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) [1] is a multi-carrier technique, which is effective and active in 4G communication systems. OFDM divides the available bandwidth into many sub-bandwidths. In OFDM, the data is divided into several streams of low rates, and the parallel data are modulated simultaneously, using multiple carriers. Due to this parallel transmission, the symbol duration increases, thus decrease the prorated amount of dispersion in time resulting from the multipath delay spread. OFDM has several advantages that makes it widely used for many communication systems such as Worldwide Interoperability for Microwave Access (WiMAX), Terrestrial Digital Video Broadcasting (DVB-T), Digital Audio Broadcasting (DAB), IEEE 802.11a standard for Wireless Local Area Networks (WLAN) and IEEE 802.16a standard for Wireless Metropolitan Area Networks (WMAN) [2]. OFDM is also used for wire-line applications, such as power line communication (PLC) and digital subscriber lines (DSL) [3]. OFDM has the ability to minimize the multi-path propagation effects and the impulse noise, because it effectively transforms the frequency selective fading channel into a flat fading channel. In addition, OFDM eliminates the need for equalizers and uses modern digital signals processing techniques, such as the fast Fourier transform (FFT) technique. Unfortunately, some major drawbacks are still associated with the OFDM transmission technique. One of these drawbacks is the high peak-to-average-power-ratio (PAPR) of transmitted OFDM signals [4]. For an OFDM signal, consisting of N individual and independent data symbols, when the N signals add up to the same phase, a substantial increase in the PAPR is observed. The value of the observed instantaneous PAPR might reach as high values as N times of the average OFDM symbol amplitude [2]. In this case, the high power amplifier (HPA) and the digital to analog converter (DAC) need large dynamic ranges to avoid the clipping of the observed large amplitude of the OFDM symbol. On the other hand, adapting the dynamic range of the HPA and the DAC to the high PAPR values increases the power consumption as well as the implementation complexity of the transceiver design. Therefore, the PAPR of OFDM signals should be reduced as long as an efficient and economic operation of the entire OFDM signal processing circuitry is desired.

Companding the ranges of largely swinging signals is one of the most popular PAPR reduction techniques in OFDM-based systems due to their low implementation complexity, low processing power, and low memory bandwidth requirements [4]. It should be highlighted that, most of the companding-based PAPR reduction techniques are based on the companding transforms that have been commonly employed in the non-uniform quantization of digitally converted analog signals. In this paper, the impact of two standard companding techniques, namely, the A-law and the μ-law techniques, on the PAPR of OFDM signals is investigated. In addition, the performances of these techniques are compared to the non-linear error function (NERF) and the absolute exponential (AEXP) techniques, with the bit error rate (BER) and the PAPR as the performance comparison metrics of interest.

The rest of this paper is organized as follows. Section II overviews the general mathematical models of OFDM-based systems. In Section III, a definition of the PAPR with mathematical form is presented. Section IV overviews companding techniques including: $A-$law, $\mu-$law, absolute exponential (AEXP) and non-linear error function (NERF). Section V presents numerical simulation results that compare the four companding techniques considered in this work with some design guidelines that helps OFDM system designers to select the best companding technique that meets their design constraints.

II. OFDM SYSTEM MODEL

Fig. 1 shows a typical block diagram of an OFDM transceiver chain that incorporates companding/decompanding to reduce the PAPR. Firstly, the input data are mapped by using various mapping schemes, such as the M-ary phase shift keying (PSK) and the quadrature amplitude modulation (QAM). Then, they are converted from serial to parallel (S/P).

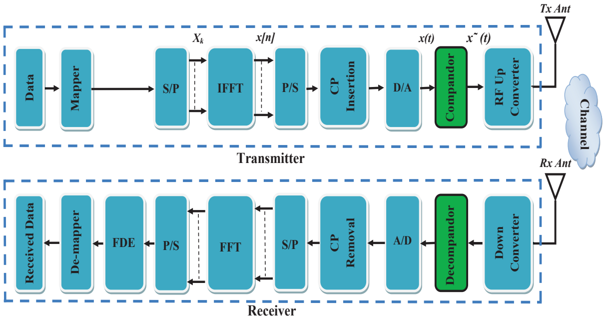

Fig. (1) Block diagram of an OFDM system, including the companding scheme. FDE: frequency domain equalizer.

After that the mapped data X_k is processed by an inverse discrete Fourier transform (IDFT). OFDM modulator transmits a large number of narrow-band carriers, closely spaced in the frequency domain. In order to avoid a large number of modulators at the transmitter and large number of de-modulators at the receiver, it is desirable to be able to use modern digital signal processing techniques, such as fast Fourier Transform (FFT). The FFT algorithm introduces an effective way to perform the DFT and the IDFT. It minimizes the number of complex multiplications from N^2 to $N/2 * log_2(N)$ and reduces the number of complex additions from $N(N-1)$ to $N.log_2(N)$ for an N-point DFT or IDFT. A parallel-to-serial (P/S) converter is applied to the resulting time domain symbols $x(n)$. A cyclic prefix (CP) of a suitable length is added to combat the effect of multipath propagation such as inter-symbol interference (ISI). Assuming that X_k; $\{k = 0, 1, ..., N-1\}$, are the complex symbols, these complex symbols are passed to the IDFT block, such that the useful time-domain OFDM symbol is obtained. The base-band OFDM signal, denoted by $x(t)$, is expressed as follows [1]:

$$x(t) = \sum_{k=0}^{N-1} X_{n,k}.e^{j2\pi k \Delta f t} \quad ; \quad 0 \leq t \leq T_s \tag{1}$$

where Δf, T_s and N represents the sub-carrier spacing, the symbol duration, and the number of sub-carriers that constitute the OFDM signal, respectively. The symbol duration should be long enough, such that $T_s \Delta f = 1$ (which is also called the condition of orthogonality), to enable the receiver to demodulate OFDM signal and get the desired information. The cyclic prefixed signal is passed through the digital-to-analog (D/A) converter to obtain the continuous-time OFDM signal $x(t)$ [5]. To avoid the clipping-induced non-linearities introduced by the HPA, a companding operator, denoted by $f(x)$, is applied to the signal $x(t)$ as follows:

$$\tilde{x}(t) = f\big(x(t)\big) \tag{2}$$

Finally, the signal is amplified using power amplifier to the desired power level and is transmitted over the communications channel [3]. As shown in Fig. 1, at the receiver side, the transmitter processing is functionally reversed in the reverse order to obtain an estimated form of the binary information sequence.

III. PEAK-TO-AVERAGE-POWER-RATIO (PAPR)

The PAPR of an OFDM signal $x(n)$ is the ratio of the peak instantaneous power to the average power of an OFDM signal. Mathematically, the PAPR, measured in dB, is expressed as follows [4]:

$$PAPR\{x(n)\}(dB) = 10.\log_{10}\left(\frac{\max|x(n)|^2}{\mathbf{E}\{|x(n)|^2\}}\right) \tag{3}$$

where, $\mathbf{E}\{.\}$ is the statistical expectation operator, P_{peak} is the peak OFDM signal power and P_{avg} is the average power of the OFDM signal.

IV. COMPANDING-BASED PAPR REDUCTION TRANSFORMS

This section overviews four companding transforms that are commonly applied to the non-uniform quantization in the ADC of analog signals.

A. A-law Companding

In A-law companding, uniform quantization is achieved at $A = 1$, where the characteristic curve is linear and no compression is conducted. For $A$is greater than one (A >1), the characteristic of the curve becomes non-linear. The practically used value of A is 87.6 as defined by the Consultative Committee for International Telephony and Telegraphy (CCITT). Mathematically the A-law companding function is defined as [6]:

$$f(x) = \begin{cases} \dfrac{A|x|}{1+lnA} sgn(x) & ; 0 < |x| \leq \dfrac{V}{A} \\ V\dfrac{1+ln\Big(A\dfrac{|x|}{V}\Big)}{1+lnA} sgn(x) & ; \dfrac{V}{A} < |x| \leq V \end{cases} \tag{4}$$

where A is the value of A-law parameter of the compressor, V is the peak signal magnitude for the signal x and $sgn(x)$ is the sign function.

B. μ-law Companding

In μ-law companding, uniform quantization is achieved at $\mu = 0$, where the characteristic curve is linear and no compression is applied, for $\mu$is greater than zero (μ >0), the characteristic curve becomes non-linear. The practically used value of $\mu = 255$ as defined by the CCITT. Mathematically, the μ-law companding function is given by [7]:

$$f(x) = V\frac{ln(1+\mu\frac{|x|}{V})}{ln(1+\mu)} sgn(x) \tag{5}$$

C. Absolute Exponential (AEXP) Companding

The AEXP companding minimizes the PAPR of companded signals by sending the statistics of the amplitudes of these signals into uniform distribution. The equation of AEXP is derived from Trapezoidal power companding and exponential companding [8]. The AEXP companding relationship is given by the following:

$$f(x) = sgn(x)\left(\alpha\left(1-\exp\left(-\frac{|x|^2}{\sigma^2}\right)\right)\right)^{1/d} \tag{6}$$

where d is the degree of companding scheme, σ^2 is the variance of the input signals and α is a positive constant that defines the average power of the output signals. To maintain the same average power level for input and output signals, α should be set as follows:

$$\alpha = \left(\frac{\mathbf{E}\{|x|^2\}}{\mathbf{E}\left\{ \left(1 - \exp\left(-\frac{|x|^2}{\sigma^2}\right)\right)^{2/d} \right\}} \right)^{d/2} \tag{7}$$

It should be noted that, in all of the afore-mentioned companding techniques, the non-linear function $f(x)$ operates on the magnitude of the baseband signal x. The phase of x is intentionally discarded to avoid further distortions impressed to the OFDM signal when including the phase to the companding operation [9].

D. Non-linear Error Function (NERF) Companding

NERF technique is suggested to minimize the high PAPR and is based on the Gaussian error function (erf). On the transmitter side, the NERF companding characteristics are expressed as [10]:

$$f(x) = k_1.erf(\frac{|x|}{\sqrt{2}\sigma})sgn(x) \tag{8}$$

where, k_1 is a positive constant that defines the average power of the output signals. To achieve power conservative companding, the value of k_1 should be set to $\sqrt{3}\sigma$.

V. SIMULATION RESULTS AND DISCUSSION

This section presents numerical simulation results for the OFDM system using the companding/de-companding techniques, with the PAPR, BER, power spectral density (PSD) and the average power as the performance metrics of interest. The randomly generated input data are modulated by Quadrature Phase Shift Keying (QPSK). The number of sub-carriers N = 512, the CP length is 128, sub-carrier spacing Δf= 150 Hz, sampling rate f_s= 9.6 kHz. The HPA is solid state power amplifier (SSPA) [11] with random positive integer parameter of SSPA, p = 1 in this paper. Urban (COST207) with 6 paths is used as a multipath channel. Table I shows the channel power and delay according to COST207 [COS89].

TABLE (I) Channel Power-Delay profile [12].

Tap number	1	2	3	4	5	6
Power (dB)	-3	0	-2	-6	-8	-10
Delay (μs)	0	0.2	0.5	1.6	2.3	5

Fig. 2 plots the I/O characteristics of the four companding transforms considered in this work. As clear from this figure, for the $A-$law and the $\mu-$law techniques, weakly swinging signals are enlarged by increasing the values of A and μ, respectively. Accordingly, the resulting companded signals possess high average power levels. However, the increase in the average power of the companded OFDM signals is accompanied by degradation in the BER performance as shown in Fig. 3 (b). AEXP and NERF companding techniques enlarge small signals and compress the peak of the signal and this leads to PAPR improvement, but at the expense of degradation in BER. Fig. 3 shows the PAPR, BER and PSD for four different companding techniques, which compares companded with uncompanded signal. In order to get a fair

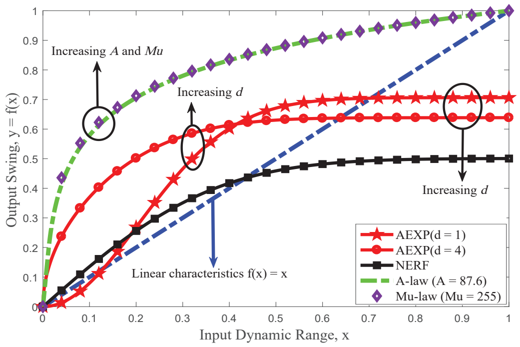

Fig. (2) The input/output characteristics of the AEXP, NERF, A-law and μ-law companding techniques.

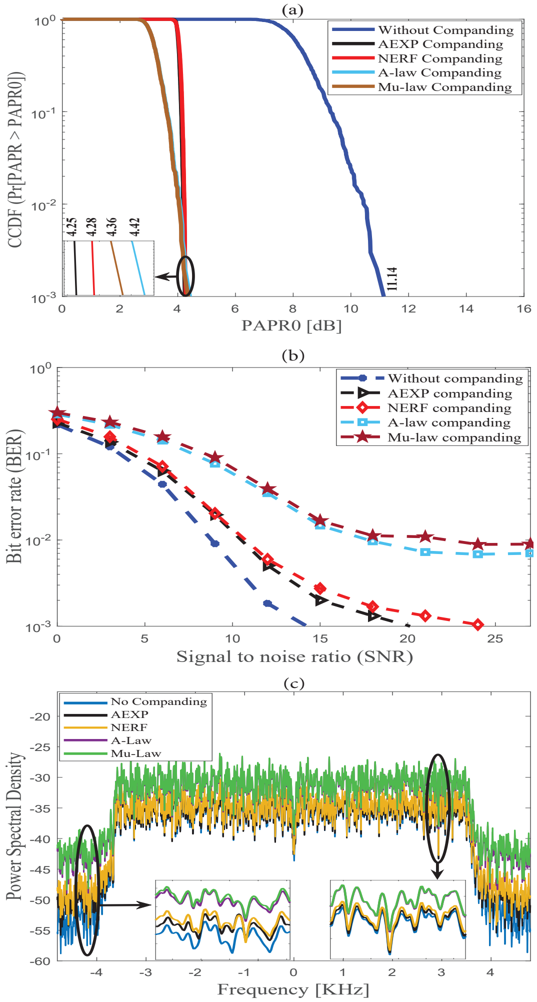

Fig. (3) Original signal versus different companding techniques with (a): PAPR, (b): BER and (c): PSD.

comparison, the parameters for all techniques are changed to obtain close results in PAPR for all techniques, as shown in Fig. 3 (a), where $A = 15.6$, $\mu = 32.5$ and $d = 1.3$. According to these parameters, the degradation in BER for all techniques is portrayed in Fig. 3 (b). Table II shows that the best improvement in PAPR (*Improvement in PAPR = 6,9 dB*) with less degradation in BER (*SNR at BER* 10^{-3} *degradation = 5.8 dB*) is achieved with AEXP companding technique. As shown in Fig. 3 (c), AEXP companding technique has much less effect on the original power spectrum comparing to the other companding techniques (NERF, A-Law and mu-Law). AEXP companding technique compresses the large signals and enlarges the small signals simultaneously, which leads to maintain the average power before and after companding unchanged; this is the main cause that AEXP companding technique has less effect on the original power spectrum.

Fig. 4 displays the comparison between companded and uncompanded signal according to average power. The average power of the OFDM signal is calculated with and without companding. It is observed that the average power for AEXP companding technique ($Avg = 0.3732$) is the closest one to the average power of original signal (0.3614), as shown in Fig. 4, and this is the reason why the AEXP companding technique is the best one for the improvement of PAPR and the less degradation in BER.

TABLE (II) Performance Comparison of Companded and Uncompanded Signals

Different Companding Techniques					
Methods	Linear	AEXP	NERF	A-law	μ-law
Parameter	—	d=1.3	NERF	A=15.6	μ=32.5
PAPR(dB) at 10^{-3}	11.14	4.25	4.28	4.42	4.36
SNR(dB) at BER 10^{-3}	14.2	20	23.9	> 27	> 27
Improvement in PAPR (dB)	—	6.89	6.86	6.72	6.78
Degradation in BER (dB)	—	5.8	9.7	> 12.8	> 12.8

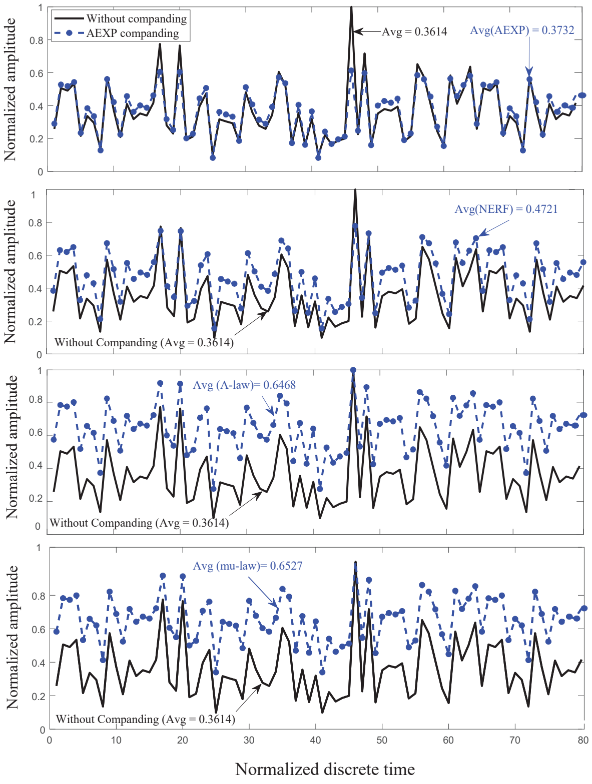

Fig. (4) Simulated time domain plots for a typical OFDM symbol with and without companding.

VI. CONCLUSION

In this paper, the performances of four typical low complexity and bandwidth efficient companding techniques (NERF, AEXP, A-law, and μ-law) are compared, with the PAPR, BER, PSD and the average power as the performance metrics of interest. Interestingly, simulation results show that there always exists a trade-off between both performance metrics. Moreover, it is observed that, the AEXP-based companding outperforms the rest of the considered techniques in terms of the adopted performance comparison metrics.

ACKNOWLEDGEMENT

This work was partially funded by ONE Lab at Zewail City of Science and Technology and at Cairo University, NTRA, ITIDA, ASRT, and NSERC.

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