Performance Evaluation of RZF Precoding in Multi-User MIMO Systems

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***Abstract*—In this paper, the transmission performance of the downlink of linearly precodied multi-user multiple input-multiple output (MIMO) systems is investigated. The zero forcing (ZF), regularized ZF (RZF) and minimum mean squared error (MMSE) precoding schemes are considered, and the bit error rate (BER) is adopted as the performance metric of interest. Simulation results indicate that, an adequate BER performance is achieved by the proper control of the design parameters of RZF precoding. Moreover, a relationship between the number of users' antennas and the BS is observed, such that an adequate BER performance is achieved.**

***Index Terms*—Multiple input-multiple output (MIMO), minimum mean squared error (MMSE), regularized zero forcing (RZF), zero forcing (ZF).**

I. INTRODUCTION

The massive multiple input- multiple output (MIMO) technology has been inspired from the fundamental concept of the conventional multi-user MIMO communication systems to enable the simultaneous time/frequency access to wireless services for tens, or even hundreds, of users. However, increasing the number of antennas in these systems is inevitably followed by a corresponding increase in the power consumed along the radio frequency (RF) up/down link chains.

The power consumed during the data transmission over the downlink that connects a base station (BS) to a user equipment (UE) in multi-user massive MIMO systems is contributed by two main processes; which are: 1- The power consumed due to the wireless transmission of signalling waveforms over fading channels, and 2- The circuit processing applied to these signals, including the linear MIMO precoding processes. The main goal of precoding the users' information at the BS transmitter side is to mitigate the inter-symbol interference (ISI) caused by the multi-path propagation channels as well as the possible multiple user interference (MUI) accumulated at the UE front end. To achieve this goal in MIMO systems, linear MIMO precoding schemes are usually employed [1]. Typically, there exist three main linear MIMO precoding schemes; the zero forcing (ZF), the minimum mean squared error (MMSE) and the regularized ZF (RZF) precoding. Throughout the literature, the ultimate performance of linearly precoded MIMO systems has been characterized using several performance evaluation metrics such as the area throughput, the energy efficiency, the computational complexity, the outage probability and the bit error rate (BER). Despite its optimal performance, the impractically high computational complexity associated with power allocation (PA) algorithms in MMSE precoding limits its application to MIMO systems in dense multi-user environments, especially in the massive MIMO regime. On the other hand, the sub-optimal performance shown by ZF precoding is attributed to the lack of PA algorithms in this scheme.

According to [6], RZF precoding shows a close-to-optimal performance without the need for the PA algorithms, provided that its regularization parameter is properly selected. Moreover, in RZF precoding, the regularization parameter is usually fixed at a pre-specified optimal value and is not adapted to the time varying SINR. Due to its adequate trade-off between the capabilities and the limitations of the ZF and MMSE schemes, RZF precoding has received a substantial part of the research interests in linearly precoded MIMO systems. Several approaches [1]-[9] have reported complexity efficient mathematical formulations of the RZF precoding technique. Some of these approaches (e.g., [3]-[5]), have been concerned with the RZF matrix inversion problem. However, to the best of the authors' knowledge, the BER performance of RZF precoded MIMO systems has not been reported as a technique that stems from the MMSE precoding such as the ZF technique.

Throughout the analysis presented in this paper, a general mathematical framework is developed to include the ZF and the precoding schemes as special cases that stem from of the MMSE precoding scheme. Moreover, a conservation relationship between the number of antennas at the BS and the UEs sides is observed from the simulated BER performance, considering RZF precoding.

The rest of this paper is organized as follows. A typical linearly precoded multi-user MIMO system is overviewed in Section II, considering the ZF, the RZF, and the MMSE precoding schemes. In Section III, the BER performance of the system described in Section II is numerically evaluated. The conclusion of the whole paper is finally presented in Section IV.

Notation: Throughout the rest of this paper, $(.)^H$ denotes the Hermitian operator, while bold face letters and symbols denote vectors and matrices.

II. LINEARLY PRECODED MULTI-USER MIMO SYSTEMS: A MODEL OVERVIEW

This section overviews ZF, RZF and MMSE precoding in the downlink of a typical multi-user MIMO system. Fig. 1

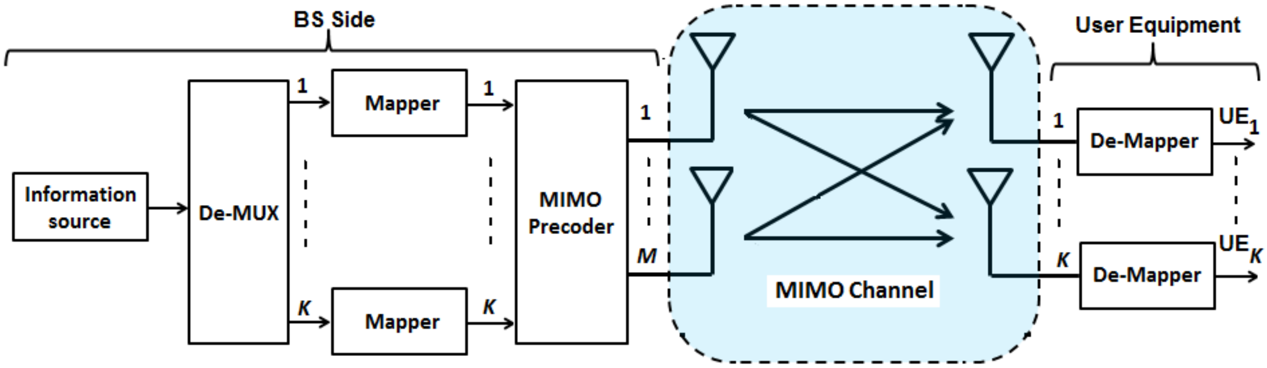

Fig. 1. Block diagram representation of a typical multi-user MIMO system that employs linear precoding schemes.

illustrates a block diagram representation of this system. As clear in this figure, the system consists of a BS that is equipped with M transmit antennas that serve K single antenna UEs uniformly deployed over the entire coverage area of the BS. At the BS side, an information source emits a sequence of independent and identically distributed (i.i.d) binary valued random variables (RVs) at a rate of R_b bps. The multiplexer (MUX) that follows the information source splits the emitted information symbol stream into K parallel streams of R_b/K bps rate each. Each of the K MUX outputs is forwarded to an $\bar{M}$-ary order constellation mapper. The samples at the output of each of the K constellation mappers are released at at symbol rate of $R_s = R_b/K\bar{M}$, where the complex sequence $\tilde{\mathbf{x}}^{(i)} \in \mathbf{C}^{K\times 1}; \tilde{\mathbf{x}}^{(i)} = [\tilde{x}_1^{(i)}, \tilde{x}_2^{(i)} ... \tilde{x}_K^{(i)}]^T$ and $\tilde{x}_k^{(i)}$ represents the i^{th} time domain information symbol of the k^{th} UE which is applied to the relevant input ports of a $K \times M$ linear precoder. The fading and multiple user interference (MUI) effects experienced by each of the K UE signals over the wireless channel are compensated *a priori* (i.e., before being emitted to the wireless channel via the M BS TX antennas), by applying a linear MIMO precoding matrix, denoted by $\tilde{\mathbf{V}}$, to $\tilde{\mathbf{x}}^{(i)}$. This is expressed mathematically as follows:

$$\tilde{\mathbf{X}}^{(i)} = \tilde{\mathbf{V}}\tilde{\mathbf{x}}^{(i)} \tag{1}$$

where $\tilde{\mathbf{X}}^{(i)} \in \mathbf{C}^{M\times 1}; \tilde{\mathbf{X}}^{(i)} = [\tilde{X}_1^{(i)}, \tilde{X}_2^{(i)} ... \tilde{X}_M^{(i)}]^T$ is the linearly precoded information vector of the K UE signals, $\tilde{\mathbf{V}} \in \mathbf{C}^{M\times K}$ is the linear precoding matrix (to be defined later in this section). Assuming perfect BS-UE synchronization, at each UE receiver front-end, the received precoded vector is corrupted by a zero mean additive white Gaussian noise (AWGN) process as follows:

$$\tilde{\mathbf{Y}}^{(i)} = \tilde{\mathbf{H}}\tilde{\mathbf{V}}\tilde{\mathbf{x}}^{(i)} + \tilde{\mathbf{N}}^{(i)} \tag{2}$$

where $\tilde{\mathbf{H}} \in \mathbf{C}^{K\times M}$ is the true CSI matrix whose entries, denoted by $\tilde{H}_{k,m}$, are sample realizations of a Rayliegh distributed random process and $\tilde{\mathbf{N}}^{(i)} \in \mathbf{C}^{K\times 1}$ is a sample realization of the AWGN process. The received vector is then applied to a constellation de-mapper such that the relevant binary data stream is obtained at the output of each UE. At the output end of the k^{th} UE, the BER performance is calculated as follows:

$$\begin{aligned} P_b &= \frac{1}{\bar{M}} \Pr\left\{\tilde{\mathbf{Y}}^{(i)} \neq \tilde{\mathbf{x}}^{(i)}\right\} \\ &\simeq \frac{1}{\bar{M}L} \sum_{i=1}^{L} \left((\tilde{\mathbf{Y}}^{(i)})_k \neq (\tilde{\mathbf{x}}^{(i)})_k \right); L \gg 10^3 \end{aligned} \tag{3}$$

where $\Pr\left\{\tilde{\mathbf{Y}}^{(i)} \neq \tilde{\mathbf{x}}^{(i)}\right\}$ yields the symbol error rate performance and L is the frame length (measured in bits) over which the classical definition of the bit error probability approximates the numerical BER values resulting from (4).

The generic precoding matrix representation in (2) that includes the three considered precoding schemes is represented as follows:

$$\tilde{\mathbf{V}} = \begin{cases} \mathbf{V}_{MMSE} & ;\text{for MMSE precoding} \\ \mathbf{V}_{RZF} & ;\text{for RZF precoding} \\ \mathbf{V}_{ZF} & ;\text{for ZF precoding} \end{cases} \tag{4}$$

For MMSE precoding, the definition of $\tilde{\mathbf{V}}$ is given by

$$\mathbf{V}_{MMSE} = \beta\left(\widehat{\mathbf{H}}^H \Gamma \widehat{\mathbf{H}} + \sigma_n^2 \mathbf{I}_M\right)^{-1} \widehat{\mathbf{H}}^H \tag{5}$$

where the parameter β is a controllable scaling constant that adapts the total BS transmitted power, $\widehat{\mathbf{H}} \in \mathbf{C}^{K\times M}$ is the estimated CSI matrix whose entries, denoted by $\widehat{H}_{k,m}$, are sample realizations of a Rayliegh distributed random process. $\Gamma = diag(p_1, p_2 ..., p_K); \Gamma \in \mathbf{R}^{K\times K}$ is the power allocation (PA) matrix, p_k is the power allocated by the BS to the k^{th} UE, $\mathbf{I}_M$ is a $M \times M$ identity matrix, and $\sigma_n^2 = \mathbf{E}\{\tilde{\mathbf{N}}^{H(i)}\tilde{\mathbf{N}}^{(i)}\}$ is the power of the AWGN process. According to [12], equal PA is assumed when RZF precoding is employed. In this context, for simplicity, $\Gamma = \mathbf{I}_M$ (i.e., no PA is employed), and σ_n^2 is replaced by a constant α, where α is a regularization parameter whose value is fixed during the transmission. Accordingly, the RZF precoding matrix is given by

$$\mathbf{V}_{RZF} = \beta\left(\widehat{\mathbf{H}}^H \widehat{\mathbf{H}} + \alpha \mathbf{I}_M\right)^{-1} \widehat{\mathbf{H}}^H \tag{6}$$

Based on the generic mathematical reformulation of (6), the ZF precoding scheme is easily obtained by substituting $\alpha = 0$ into (7). This substitution yields the following ZF precoding matrix:

$$\mathbf{V}_{ZF} = \beta\left(\widehat{\mathbf{H}}^{H}\widehat{\mathbf{H}}\right)^{-1}\widehat{\mathbf{H}}^{H} \tag{7}$$

It should be highlighted that, as clear from (6), the MMSE precoder accounts for the particular operating SINR value of each UE due to the PA matrix Γ as well as the AWGN power. According to [12], the complexity of calculating a convergent form of the PA matrix is about three times the complexity of initializing this matrix. On the other hand, as clear from (8), the ZF precoding scheme does not account for the variations of the SINR and/or the variations of the AWGN power as seen at each UE front-end. The RZF precoder has the advantage of being independent of the power allocated for each UE. Moreover, the RZF precoding scheme preserves a constant value of α, regardless of the variation in σ_n^2. Both of these afore-mentioned advantages recommends the RZF precoding as a feasible alternative to the ZF and the MMSE precoding schemes.

III. Simulation Results and Analysis

This section presents numerical simulation results for the BER performance of the linearly precoded multi-user MIMO system discussed in Section II. For simplicity and without loss of generality, quadrature phase shift keying (QPSK) constellation mapping is assumed for all UEs. The BER performance is evaluated based on (4), using the Monte-Carlo simulation technique, by averaging the QPSK symbol errors over a number of $L = 10^4$ symbols. In addition, throughout the simulations, it is assumed that perfect channel state information is available at the BS side (i.e., $\widehat{\mathbf{H}} = \tilde{\mathbf{H}}$). The simulation starts by generating the channel matrix $\tilde{\mathbf{H}}$ and then by calculating the precoding matrices in (6), (7), and (8).

Fig. 2 plots the BER performance versus the regularization parameter α for an RZF precoded 3×5 MIMO system. As clear from this figure, increasing the value of α leads to a deterioration in the end-to-end BER performance, regardless of the value of the SNR. Moreover, the BER is upper bounded by a value of 0.3734, corresponding to α_{max} of 10^2, regardless of the operating SNR value. On the other hand, there exists a minimum value of $\alpha_{min} = 10^{-2}$ below which a bounding BER floor is observed. For $\alpha \geq \alpha_{max}$ the BER performance becomes essentially independent of the particular value of α and the SNR, whereas for $\alpha \leq \alpha_{min}$, the BER is dependent solely on the value of the operating SNR. The BER performance is sensitive to the variations in the value of α over only a limited range, bounded by α_{min} and α_{max}. It can be correctly concluded that, in practical RZF precoded multi-user MIMO systems, the particular value of α is determined by an adequately pre-specified BER level.

In this context, α_{min} is adopted as the most proper value for RZF precoding. Fig. 3 compares the average BER performance for the RZF precoding at different values of its regularization parameter α to that of the ZF and the MMSE precoding schemes. Obviously, the higher is the value of α, the more degradation is observed in the BER performance of RZF precoding. A BER performance worse or better thann the ZF and the MMSE schemes is observed, respectively, depending on the particular value of the regularization parameter. This remark leads to the conclusion that hopping between different values of α should be performed at the BS side if a maximum BER performance is desired.

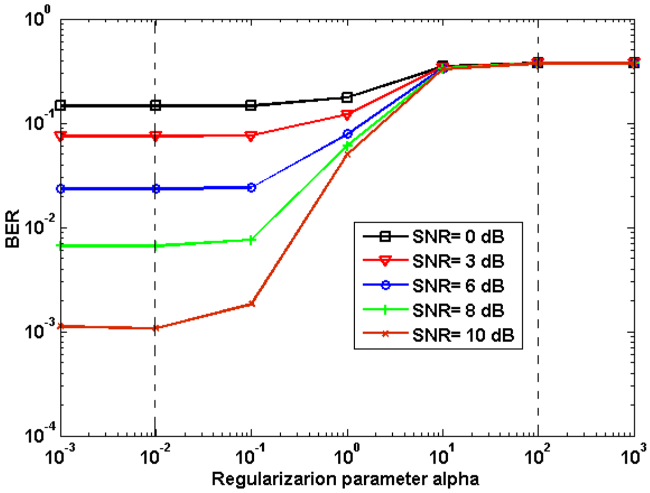

Fig. 2. BER versus the regularization parameter α of the RZF precoding scheme.

Fig. 4 depicts the BER performance versus the number of BS antennas, M, and the number of single antenna UEs, K, with RZF precoding at α_{min}, while Figs. 5 (a) and (b) illustrate the projections of the BER surface of Fig. 4 on the BER-K and the BER-M planes, respectively. As clear from Fig. 4, there exists a clear and sharp boundary, defined by the linear relationship $M = K$, between the high and the low BER regions. This boundary condition is shown as the white border line in Fig. 4. For $M < K$, the value of the BER strongly depends on the particular value of the (M, K) pair, while for $M > K$, the BER performance is essentially independent of (M, K) and is fixed at a value of about 7×10^{-3}. In Figs. 5 (a), as expected, increasing the number of UEs leads directly to the deterioration of the BER performance due to the increased number of multiple-user interferers. This performance degradation is compensated by increasing the number of serving antennas at the BS side such that the BER is improved as shown in Fig. 5 (b).

IV. CONCLUSION

In this paper, the bit error rate performance of a $K \times M$ multi-user MIMO system is investigated, considering the ZF, RZF, and the MMSE linear precoding schemes. Simulation results indicate that, an RZF precoded multi-user MIMO systems outperforms ZF and MMSE corresponding precoded systems in terms of the bit error rate performance.

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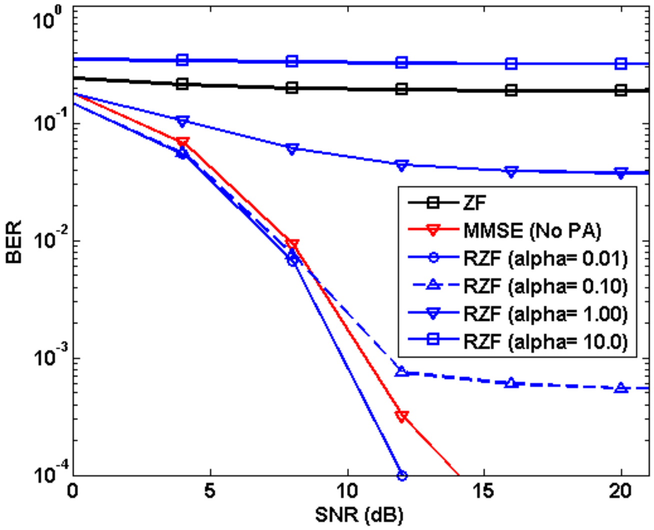

Fig. 3. BER versus SNR performance of multi-user MIMO system with ZF, RZF and MMSE precoding schemes.

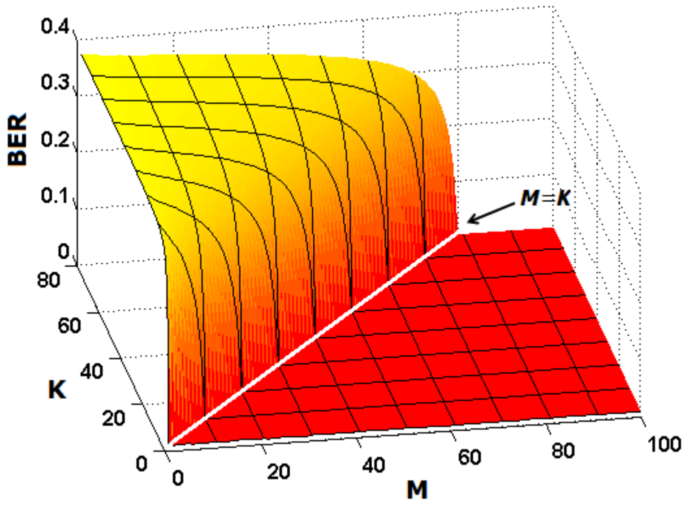

Fig. 4. BER performance versus the number of BS antennas, M, and the number of single antenna UEs, K, with RZF precoding at α_{min}.

by SRC, in part by ASRT, in part by NTRA, and in part by MCIT.

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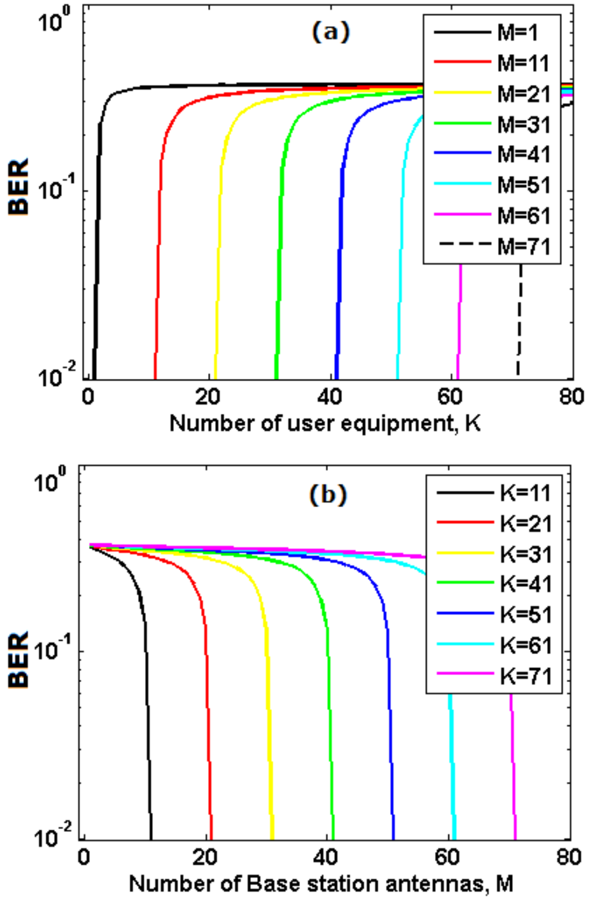

Fig. 5. BER performance of RZF precoded MIMO systems versus (a): the number of single antenna UEs, K, at selected values of the number BS antennas and (b): the number of BS antennas, M, at selected values of the number single antenna UEs.

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